

## Misfit Stresses Due to a Cylindrical Dilatational Inclusion of Annular-Sector Cross-Section in an Infinite Elastic Medium

Zh.V. Gudkina<sup>1,2</sup>, S.A. Krasnitskii<sup>1,3,4</sup> and M.Yu. Gutkin<sup>1,3,4</sup>

<sup>1</sup>ITMO University, Kronverksky 49 A, St. Petersburg, 197101, Russia

<sup>2</sup>Ioffe Physical-Technical Institute, Russian Academy of Sciences, Polytekhnicheskaya 26, St. Petersburg, 194021, Russia

<sup>3</sup>Institute for Problems in Mechanical Engineering, Russian Academy of Sciences, Bolshoi 61, Vasil. Ostrov, St. Petersburg, 199178, Russia

<sup>4</sup>Peter the Great St. Petersburg Polytechnic University, Polytekhnicheskaya 29, St. Petersburg, 195251, Russia

Received: December 10, 2021

Corresponding author: [Zh.V. Gudkina](#)

**Abstract.** An elastic model for a cylindrical dilatational inclusion of annular-sector cross-section in an infinite elastic medium is considered. The stress fields are found in a closed analytical form and are illustrated by stress maps. Specific features in the stress distribution are revealed and discussed in detail. It is shown that the stress magnitude can be so high that various mechanisms of stress relaxation can be activated.

### REFERENCE LIST

- [1] K. Zhou, H.J. Hoh, X. Wang, L.M. Keer, J.H. Pang, B. Song and Q.J. Wang, *A review of recent works on inclusions*, Mech. Mater., 2013, vol. 60, pp. 144–158.
- [2] J.D. Eshelby, *The determination of the elastic field of an ellipsoidal inclusion, and related problems*, Proc. R. Soc. Lond. A, 1957, vol. 241, no. 1226, pp. 376–396.
- [3] B.N. Kuvshinov, *Elastic and piezoelectric fields due to polyhedral inclusions*, Int. J. Solids Struct., 2008, vol. 45, no. 5, pp. 1352–1384.
- [4] R.D. Mindlin, *Force at a point in the interior of a semi-infinite solid*, Physics, 1936, vol. 7, no. 5, pp. 195–202.
- [5] J.N. Goodier, *On the integration of the thermoelastic equations*, Phil. Mag., 1937, vol. 23, no. 157, pp. 1017–1032.
- [6] R.D. Mindlin and D.H. Cheng, *Thermoelastic stress in the semi-infinite solid*, J. Appl. Phys., 1950, vol. 21, no. 9, pp. 931–933.
- [7] B. Sen, *Note on stresses produced by nuclei of thermo-elastic strain in a semi-infinite elastic solid*, Quatr. Appl. Math., 1951, vol. 8, no. 4, pp. 365–369.
- [8] J.A. Geertsma, *A remark on the analogy between thermoelasticity and the elasticity of saturated elastic porous media*, J. Mech. Phys. Sol., 1957, vol. 6, no. 1, pp. 13–16.
- [9] J.A. Steketee, *On Volterra's dislocations in a semi-infinite elastic medium*, Can. J. Phys., 1958, vol. 36, no. 2, pp. 192–205.
- [10] L. Rongved and J.T. Frazier, *Displacement discontinuity in the elastic half-space*, J. Appl. Mech., 1958, vol. 25, no. 1, pp. 125–128.

- [11] J. Geertsma, *Land subsidence above compacting oil and gas reservoir*, J. Pet. Technol., 1973, vol. 25, no. 6, pp. 734–744.
- [12] W. Nowacki, *Thermoelasticity*, Pergamon, Oxford, 1986.
- [13] T. Mura, *Micromechanics of Defects in Solids*, Martinus Nijhoff Publishers, Dordrecht, 1987.
- [14] M. Kachanov and I. Sevostianov, *Micromechanics of Materials, with Applications*, Springer, 2018.
- [15] M. Doi, *Elasticity effects on the microstructure of alloys containing coherent precipitates*, Prog. Mater. Sci, 1996, vol. 40, no. 2, pp. 79–180.
- [16] X. Zhao, R. Duddu, S.P.A. Bordas and J. Qu, *Effects of elastic strain energy and interfacial stress on the equilibrium morphology of misfit particles in heterogeneous solids*, J. Mech. Phys. Solids, 2013, vol. 61, no. 6, pp. 1433–1445.
- [17] A.P. Chernakov, A.L. Kolesnikova, M.Yu. Gutkin and A.E. Romanov, *Periodic array of misfit dislocation loops and stress relaxation in core-shell nanowires*, Int. J. Eng. Sci., 2020, vol. 156, art. no. 103367.
- [18] S. Krasnitskii, A. Trofimov, E. Radi and I. Sevostianov, *Effect of a rigid toroidal inhomogeneity on the elastic properties of a composite*, Math. Mech. Solids., 2019, vol. 24, no. 4, pp. 1129–1146.
- [19] F. Glas, *Strain in nanowires and nanowire heterostructures*, Semiconductors and Semimetals, 2015, vol. 93, pp. 79–123.
- [20] L. Lanzoni, E. Radi and I. Sevostianov, *Effect of spherical pores coalescence on the overall conductivity of a material*, Mech. Mater., 2020, vol. 148, art. no. 103463.
- [21] E. Radi and I. Sevostianov, *Toroidal insulating inhomogeneity in an infinite space and related problems*, Proc. Math. Phys. Eng. Sci., 2016, vol. 472, no. 2187, art. no. 20150781.
- [22] A.L. Kolesnikova, M.Yu. Gutkin and A.E. Romanov, *Analytical elastic models of finite cylindrical and truncated spherical inclusions*, Int. J. Solids Struct., 2018, vol. 143, pp. 59–72.
- [23] F. Glas, *Elastic relaxation of a truncated circular cylinder with uniform dilatational eigenstrain in a half space*, Phys. Status Solidi B, 2003, vol. 237, no. 2, pp. 599–610.
- [24] A.L. Kolesnikova, R.M. Soroka and A.E. Romanov, *Defects in the elastic continuum: classification, fields and physical analogies*, Mater. Phys. Mech., 2013, vol. 17, no. 1, pp. 71–91.
- [25] K.L. Malyshev, M.Yu. Gutkin, A.E. Romanov, A.A. Sitnikova and L.M. Sorokin, *Stress fields and diffraction contrast of rod-like defects in silicon*, Sov. Phys.-Solid State, 1988, vol. 30, no. 7, pp. 1176–1179.
- [26] M.Yu. Gutkin, *Elastic behavior of defects in nanomaterials. I. Models for infinite and semi-infinite media*, Rev. Adv. Mater. Sci., 2006, vol. 13, no. 2, pp. 125–161.
- [27] K.N. Mikaelyan, M.Yu. Gutkin, E.N. Borodin and A.E. Romanov, *Dislocation emission from the edge of a misfitting nanowire embedded in a free-standing nanolayer*, Int. J. Solids Struct., 2019, vol. 161, pp. 127–135.
- [28] A.M. Smirnov, S.A. Krasnitskii and M.Yu. Gutkin, *Generation of misfit dislocations in a core-shell nanowire near the edge of prismatic core*, Acta Mater., 2020, vol. 186, pp. 494–510.