

## Enhanced Ductility of High-Strength Ultrafine-Grained Aluminum Alloys at Ambient Temperature (Review)

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**Abstract.** Bulk nanostructured, or ultrafine-grained metals and alloys structured by severe plastic deformation (SPD) methods usually demonstrate high strength and reduced ductility. The poor ductility is a critical issue which limits their practical applications. Significant efforts were made to improve tensile ductility of the SPD-processed metallic materials while keeping sufficiently high strength. In this paper we present a short overview of the developed approaches for simultaneous improvement of the strength and ductility of Al-based alloys with an emphasis on the recent finding and physical reasons of the plasticity enhancement. The main attention is paid to achieving increased ductility of high strength aluminum alloy at room temperature.

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